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Radiative transition probabilities for the main diatomic electronic systems of N_2 , N_2^+ , NO, O_2 , CO, CO⁺, CN, C_2 and H_2 produced in plasma of atmospheric entry

Z. Qin^a, J.M. Zhao^a, L.H. Liu^{a,b,*}

^a School of Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001, China ^b Department of Physics, Harbin Institute of Technology, Harbin 150001, China

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ABSTRACT

Accurate radiative transition probabilities of diatomic electronic systems are required to calculate the discrete radiation of plasmas. However, most of the published transition probabilities are obtained using older spectroscopic constants and electronic transition moment functions (ETMFs), some of which deviates greatly from experimental data. Fortunately, a lot of new spectroscopic constants that include more anharmonic correction terms than the earlier ones have been published over the past few years. In this work, the Einstein coefficients, Franck-Condon factors and absorption band oscillator strengths are calculated for important diatomic radiative transition processes of N2-O2, CO2-N2 and H2 plasmas produced in entering into the atmosphere of Earth, Mars and Jupiter. The most up-to-date spectroscopic constants are selected to reconstruct the potential energy curves by the Rydberg-Klein-Rees (RKR) method. Then the vibrational wave functions are calculated through the resolution of the radial Schrödinger equation for such potential energy curves. These results, together with the latest "ab-initio" ETMFs derived from the literature are used to compute the square of electronic-vibrational transition moments, Einstein coefficients and absorption band oscillator strengths. Moreover, the Franck-Condon factors are determined with the obtained vibrational wave functions. In the supplementary material we present tables of the radiative transition probabilities for 40 band systems of N₂, N₂⁺, NO, O₂, CO, CO⁺, CN, C₂ and H₂ molecules. In addition, the calculated radiative lifetimes are systematically validated by available experimental results.

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1. Introduction

Accurate radiative transition probabilities are key to scientific research areas including plasma optical diagnostics [1,2], quantitative spectroscopy [2,3], analyzation of emission from gas discharge and afterglows [4,5], radiation modeling of atmospheric entry [6–8,58], etc. In particular, strong shock waves, formed in front of the vehicles during hypersonic flights, translate part of the kinetic energy of the vehicles into internal energy of the gases. The high-temperature gases will radiate, and the emitted radiation may also contribute to the heat flux suffered by the vehicles, which will be very important for vehicles that enter into the atmosphere at very high speed. Hence, the prediction of radiation for high-temperature gases in shock layers is required for the efficient design of thermal protection systems. The emission coefficients ε_{σ} of

http://dx.doi.org/10.1016/j.jqsrt.2017.08.010 0022-4073/© 2017 Elsevier Ltd. All rights reserved. high-temperature gases is generally given by

$$\varepsilon_{\sigma} = \sum_{ul} \frac{A_{ul}}{4\pi} hc v_{ul} N_u f_{ul} (v - v_{ul}) \tag{1}$$

where A_{ul} is the Einstein coefficients N_u denotes the population of the upper transition levels. v_{ul} is the wavenumber, and $f_{ul}(v - v_{ul})$ is the spectral line shape of the transition. Einstein coefficients, as a form of radiative transition probabilities, are vital to the determination of the emission coefficients that are utilized further to calculate the radiative heat flux.

Einstein coefficients, absorption band oscillator strength and Franck–Condon factors are three forms of the radiative transition probabilities. The first two parameters can be calculated from the square of electronic-vibrational transition moment and Franck–Condon factors are determined with the vibrational wave functions. Moreover, the square of electronic-vibrational transition moment can be calculated using Franck–Condon factors $q_{v'v''}$ (v' and v'' are vibrational levels in the upper and lower electronic levels of transition, respectively), electronic transition moment function

^{*} Corresponding author at: School of Energy Science and Engineering, Harbin Institute of Technology, 92 West Dazhi Street, Harbin 150001, China *E-mail address:* lhliu@hit.edu.cn (L.H. Liu).

(ETMF) $R_e(r)$ and r-centroids $r_{v'v''}$ in the r-centroid approximation [9,10]:

$$\left(R_{e}^{\nu'\nu''}\right)^{2} \cong \left[R_{e}(\bar{r}_{\nu'\nu''})\right]^{2} q_{\nu'\nu''} \tag{2}$$

Such approximations often need to be combined with the limited experimental data to calculate Einstein coefficients for the large number of band systems. Another more rigorous method to obtain the square of electronic-vibrational transition moment is given by

$$\left(R_{e}^{\nu'\nu''}\right)^{2} = \left[\int_{0}^{\infty} \psi_{\nu'}(r)R_{e}(r)\psi_{\nu''}(r)dr\right]^{2}$$
(3)

where *r* is the internuclear distance, $\psi_{v'}(r)$ and $\psi_{v''}(r)$ are the corresponding radial vibrational wave functions and $R_e(r)$ is the ETMF, which is an average of the electric dipole moment with respect to the electronic wave function.

Thus, in order to precisely determine the electronic-vibrational transition moment, it is necessary to obtain the parameters of the vibrational wave functions and the ETMF. The vibrational wave function can be obtained by solving the radial Schrödinger equation over the potential energy curve for a selected electronic state of a molecule. And the potential energy curves of the different electronic states of a diatomic molecule can be determined by the ab-initio method [11] or the Rydberg-Klein-Rees (RKR) method [12–14]. The ab-initio method, based on quantum chemistry theory, can predict the potential energy curves to a good level of precision, whereas the RKR method, a more straightforward method grounded on experimental spectroscopic constants, can predict potential curves more accurately for internuclear distances where the spectroscopy constants are valid. Therefore, the RKR method is chosen to reconstruct the potential curves for the efficient internuclear distances in this paper. The ETMF can be determined by band strength measurements, or obtained by quantum-mechanical calculations which have been proved to attain an accuracy comparable to that of many band intensity measurements. Besides, such calculations cover a wider range of internuclear distances than those by the r-centroid approximation method [15]. For these reasons, the relation of Eq. (3) is used to calculate the electronicvibrational transition moments taking advantage of the accurate spectroscopy constants and ab-initio ETMFs for all the band systems considered here.

In view of the important role of transition probabilities for diatomic molecular band systems of plasmas in hypersonic flights, a lot of data of transition probabilities have been published over the past few decades. Krupenie [16] gave a critical review and completion of the observed and predicted spectroscopy data of O₂ and its ions O_2^- , O_2^+ and tabulated many of these transition probabilities for several oxygen band systems. Lofthus and Krupenie [17] presented many of these transition probabilities for several band systems of N_2 and its ions N_2^- , N_2^+ . Subsequently, many papers were published regarding transition probabilities for some band systems of N₂, N₂⁺, O₂, NO, CO, CO⁺, CN, and C₂ diatomic molecules [18-25]. More recently, da Silva and Dudeck [26] calculated Franck-Condon factors, Einstein coefficients and absorption band oscillator strengths for the most prominent diatomic radiative transitions in high-temperature CO₂-N₂ plasmas. In addition, Chauveau et al. [27] constructed a spectroscopic database for all the contributing electronic systems of air diatomic molecules based on transition probabilities, whose correctness and precision had been proved by measured radiative lifetimes. A similar approach was taken by Babou et al. [28] to obtain transition probabilities of diatomic electronic band systems for CO₂-N₂ plasma, including CO, CO⁺, CN and C₂ molecules.

However, the transition probabilities are still incomplete and some of them are less accurate. Moreover, many spectroscopic constants and ETMFs are so old that can be superseded by more recent measurements and calculations. For these reasons, the potential energy curves of electronic states of N₂, N₂⁺, NO, O₂, CO, CO⁺, CN, C₂ and H₂ molecules have been reconstructed by RKR method starting from the selected most up-to-date spectroscopic constants. Then the vibrational wave functions are attained by solving the radial Schrödinger equation over the reconstructed energy potential curves. Finally, we calculate the transition probabilities for all the band systems considered using vibrational wavefunctions and ETMFs available in the literature. To validate the calculated transition probabilities, the vibrational radiative lifetimes calculated from Einstein coefficients are systematically compared with available experimental results and other calculations. This paper is organized as follows. In Section 2, we describe the details of the RKR method. The results of reconstructed potential energy curves are given in Section 3. In Sections 4 and 5, the results of transition probabilities and radiative lifetimes are given and discussed. In Section 6, conclusions are drawn.

2. Methods of calculation

The RKR method is a widely used first-order semiclassical inversion procedure for reconstructing potential energy curves of diatomic electronic states. This theory is extensively discussed in a number of literatures and monographs [12–14,29–30]. The key conclusions of this theory are made up of two Klein integrals, which are given by [29]

$$r_2(\nu) - r_1(\nu) = 2\sqrt{\frac{\hbar^2}{2\mu}} \int_{\nu_{\min}}^{\nu} \frac{d\nu'}{\left[G_{\nu} - G_{\nu'}\right]^{1/2}} = 2f(\nu)$$
(4)

$$\frac{1}{r_1(\nu)} - \frac{1}{r_2(\nu)} = 2\sqrt{\frac{2\mu}{\hbar^2}} \int_{\nu_{\min}}^{\nu} \frac{B_{\nu'} d\nu'}{\left[G_{\nu} - G_{\nu'}\right]^{1/2}} = 2g(\nu)$$
(5)

where $r_1(v)$ and $r_2(v)$ are inner and outer classical turning points of the potential curve for a given vibrational level, μ is the reduced mass of the molecule, and \hbar is the reduced Planck's constant, v_{min} is the non-integer effective value of the vibrational quantum number at the potential minimum, namely the zero of vibrational level energy G_v , B_v is the inertial rotational constant for that vibrational level, and they can be expanded as

$$G_{\nu} = \sum_{l=1}^{n} Y_{l,0} (\nu + 1/2)^{l} = \omega_{e} (\nu + 1/2) - \omega_{e} \chi_{e} (\nu + 1/2)^{2} + \cdots$$
(6)

$$B_{\nu} = \sum_{l=0}^{m} Y_{l,1} (\nu + 1/2)^{l} = B_{e} - \alpha_{e} (\nu + 1/2) + \cdots$$
(7)

where $Y_{l,0}$ and $Y_{l,1}$ are Dunham coefficients rearranged by equilibrium spectroscopic constants that are adjusted on experimental determined vibrational energy at each vibrational quantum number *v*. The most accurate available spectroscopic constants that include more anharmonic correction terms are selected. The values and references of spectroscopic constants selected for each electronic state are presented in Appendix A. Rearrangement of the Eqs. (4)–(5) yields the RKR classical turning point expressions [29]:

$$r_1(\nu) = \sqrt{\frac{f(\nu)}{g(\nu)} + f^2(\nu)} - f(\nu)$$
(8)

$$r_2(v) = \sqrt{\frac{f(v)}{g(v)} + f^2(v)} + f(v)$$
(9)

In order to remove the singularity in the integral f(v) and g(v) at v' = v and maintain high accuracy, the expressions of f(v) and g(v) have been integrated using a 128 points Gauss-type quadrature. And for the sake of calculating the vibrational wave functions, it is important to interpolate the potential energy curve between $r_1(v)$ and $r_2(v)$ and extrapolate that potential energy curve

beyond this internuclear distance. The interpolation is done with a high-order Lagrange polynomial [31]. Outside this region, the RKR potential energy curve has been extrapolated by using a repulsive function $U_{rep}(r)$ for the shorter internuclear distance and a Hulburt and Hirschfelder potential $U_{HH}(r)$ [32] for the longer internuclear distance, which can accurately give continuous single-well potential energy curve. These two functions are written as [32]

$$U_{rep}(r) = \frac{a_1}{r^{b_1}}$$
(10)

$$U_{HH}(r) = D_e \Big[1 - e^{-a_2(r-r_e)} \Big]^2 + D_e \Big[b_2 a_2^3 (r - r_e)^3 e^{-2a_2(r-r_e)} (1 + a_2 c_2(r - r_e)) \Big]$$
(11)

where D_e and r_e refer to the dissociation energy of the electronic (relative to bottom of the potential energy curve) and the equilibrium internuclear distance, respectively. a_1 , b_1 , a_2 , b_2 and c_2 are floating parameters that need to be adjusted to ensure continuity with the potential energy curve and they are adjusted in order that each extrapolation function fits a significant part of the extremity of the reconstructed potential energy curve, too.

The reconstructed potential energy curves are utilized in radial Schrödinger equation to obtain the vibrational wave functions, and the ETMFs derived from references are summarized in Section 3. The knowledge of ETMF $R_e(r)$ and vibrational wave function $\psi_v(r)$ allows determining the transition probabilities such as Einstein coefficients $A_{v'v'}$, absorption band oscillator strengths $f_{v'v'}^{abs}$, the Franck–Condon factor $q_{v'v'}$ and radiative lifetimes $\tau_{v'}$ through the following expressions [33,34]:

$$A_{\nu'\nu''} = 2.026 \times 10^{-6} \eta_{\nu'\nu''}^3 \frac{2 - \delta_{0,\Lambda'+\Lambda''}}{2 - \delta_{0,\Lambda'}} \left(R_e^{\nu'\nu''}\right)^2 \tag{12}$$

$$f_{\nu'\nu''}^{abs} = 3.0376 \times 10^{-6} \eta_{\nu'\nu''} \frac{2 - \delta_{0,\Lambda'+\Lambda''}}{2 - \delta_{0,\Lambda'}} \left(R_e^{\nu'\nu''} \right)^2 \tag{13}$$

$$q_{\nu'\nu''} = \left(\int \psi_{\nu'}(r)\psi_{\nu''}(r)dr\right)^2$$
(14)

$$\tau_{\nu'} = \frac{1}{\sum_{\nu''=0}^{\nu''\max} A_{\nu'\nu''}}$$
(15)

where $\eta_{\nu'\nu''}$ is the wavenumber of the vibrational band system. Λ' and Λ'' are the projections of the electronic orbital angular momentum on the internuclear axis for the upper and lower electronic levels, respectively.

3. Reconstructed potential energy curves

The most accurate and up-to-date spectroscopic constants and the dissociation limits, which are given in Appendix A, have been selected from the literature. The equilibrium internuclear distance r_e of the potential energy curve is selected from Ref. [40] when that is unavailable in the references given in Appendix A. The traditional spectroscopic constants are usually rearranged into Dunham coefficients. The relation between Dunham coefficients and traditional spectroscopic constants can be easily deduced from Eqs. (6)–(7). Note that v^{max} is the maximum v value of experimental measurements to adjust the potential energy curve for the extrapolations described in Eqs. (10)–(11). Namely, v^{max} is the maximum vibrational level where the Dunham coefficients are valid. Based on accurate spectroscopic constants, the reconstructed potential energy curves can be obtained by the method elaborated in Section 2. The resulting potential energy curves are potted in Fig. 1 for the 44 electronic states involved in the radiative transitions in Table 1. Furthermore, for some states, the extrapolation regions are strongly perturbed so that such extrapolations are questionable. The notations of these states are marked in red and italics in Fig.1 and the corresponding potential energy curves always show peak values.

The ETMFs for each electronic transition system are obtained from references given in Table 1. For most of the electronic transition systems, we select the "ab-initio" ETMFs to calculate the radiative transition probabilities. Some of the ETMFs determined by experiments are selected when the corresponding "ab-initio" ETMFs are not available in the literature. Details of the selected ETMFs are given in Section 5.

4. Radiative transition probabilities

Tables of radiative transition probabilities (Einstein coefficients, absorption band oscillator strengths and Franck–Condon factors) have been given in supplementary material for each electronic transition system of N_2 , N_2^+ , NO, O_2 , CO, CO⁺, CN, C_2 and H_2 diatomic molecules.

5. Vibrational radiative lifetimes

Details of the selected ETMFs for different electronic transition systems of N₂, N₂⁺, NO, O₂, CO, CO⁺, CN, C₂ and H₂ diatomic molecules and comparisons with the theoretical and experimental radiative lifetimes in the literature are given in this section. For most of electronic transition systems, our calculated results are in good agreement with the available experimental results.

5.1. N₂ transitions

For the First-positive system, Chauveau et al. [27] calculated two vibrational radiative lifetimes with the ETMFs in Refs. [63,145] and gave a list of experimental results from various references. We select the latest ab-initio ETMF of Ref. [172] to obtain radiative transition probabilities, from which the vibrational radiative lifetimes of the B³ Π_g electronic state of N₂ are calculated and shown in Fig. 2(a). Fig. 2(a) also shows different theoretical and experimental results from the references. Most of the experimental results are distributed between two calculated vibrational radiative lifetimes from Chauveau et al. [27]. Our calculated results are approximately the mean of these two results and are in good agreement with the experimental results, especially with the latest ones from [91].

For the Second-positive system, we choose the ETMF in Ref. [63]. Fig. 2(b) gives the various theoretical and experimental vibrational radiative lifetimes of the $C^3\Pi_u$ electronic state of N₂. The experimental radiative lifetimes scatter to a large extent and have a larger range of error. Our calculated results are within the allowable error range of the experimental results except for v' = 4. Radiative lifetimes from Ref. [27] are also plotted in Fig. 2(b) and these values are smaller than the present results, although they have been obtained from the same ETMF.

Concerning Vegard–Kaplan system, we select the ETMF of Ref. [87] as recommended and reported by Gilmore et al. [24]. The Einstein coefficients calculated in this work are in very good agreement with the results obtained by Gilmore et al. [15]. Spectroscopic observations have provided firm evidence for the 1.36±0.27 s lifetime of the $\Sigma = 0$ sub-state of $A^3 \Sigma_u^+$ by Shemansky and Carleton [88]. The present calculated radiative lifetime is 1.63 s, which is very close to the value observed by Shemansky and Carleton for the level $\nu' = 0$ of the $A^3 \Sigma_u^+$ electronic state.

The ETMFs selected for the Wu–Benesch, Lyman–Birge–Hopfield and IR afterglow systems are from Ref. [24], which give the ETMFs



Fig. 1. Reconstructed potential energy curves of the selected electronic states of (a) N₂, (b) N₂⁺, (c) NO, (d) O₂, (e) CO, (f) CO⁺, (g) CN, (h) C₂ and (i) H₂ molecules.

by fitting theoretical values or geometric mean of theoretical results from previous literatures. In the case of Wu–Benesch system, Fig. 2(c) shows the vibrational radiative lifetimes of the $W^3 \Delta_u$ electronic state of N₂ from this work and other references. A good agreement has been achieved between our results and the calculated results from Werner et al. [39] and Gilmore et al. [15]. With respect to Lyman–Birge–Hopfield system, a quite good agreement is observed in Fig. 2(d) between our results and the theoretical results of Ref. [101], but a deviation of about 4% is observed at v' = 1 between the present radiative lifetime and the one measured by Marinelli et al. [102]. As for IR afterglow system, it is difficult to find the experimental radiative lifetimes of the $B'^3 \Sigma_u$ electronic state of N₂, while the agreement between the present results and the calculated ones from Ref. [39] is within 5% for v' = 0-12.

5.2. N₂⁺ transitions

For the Meinel system, we choose the ETMF published by Langhoff et al. [66], and a very good agreement with the experimental radiative lifetimes from Ref. [104–106] is acquired, although the present radiative lifetimes are slightly larger than the calculated ones of Ref. [27], just as shown in Fig. 3(a).

Considering the First-negative and Second-negative systems, the ETMFs of Ref. [67] are used in this paper. We have plotted in Fig. 3(b) and (c) the present radiative lifetimes, theoretical radiative lifetimes from Ref. [27] and other two experimental values of the

 $B^2 \Sigma_u^+$ electronic state and the $C^2 \Sigma_u^+$ electronic state of N₂⁺. A larger value of radiative lifetimes are observed compared to that of Ref. [27], but our results are closer to the experimental results.

5.3. NO transitions

Fig. 4(a) shows vibrational radiative lifetimes of the $A^2\Sigma^+$ electronic state of NO. The present calculated radiative lifetimes, which are obtained from the ETMF of Ref. [68], are in very good agreement with the calculated ones reported in Ref. [27]. The experimental results are relatively scattered, whereas our calculated results are in good agreement with the experimental results for $v' \leq 2$.

In the same way as it has been used above, we have reported in Fig. 4(b) radiative lifetimes of the $B^2\Pi_r$ electronic state of NO calculated in the present work from the ETMF of Ref. [69]. They are compared with the theoretical results of Ref. [27] and different experimental results from Refs. [89,109,114–116]. Our calculated radiative lifetimes are closer to the experimental results of Gadd and Slanger [115].

For the $C^2 \Pi_r$ electronic state of NO, Brzozowski et al. [89] found the radiative lifetime of $C^2 \Pi_r$ state (v' = 0J < 4.5) is 32.0 ± 2.0 ns. While Smith and Read [90] obtained an upper limit of radiative lifetime of $C^2 \Pi_r$ state (v' = 0), which is $\tau \le 2.7$ ns (standard deviation 1.5 ns). What's more, the levels located higher than v' = 0 and

Molecule	Electronic system	Upper-lower states	Calculated band ($0:\nu'_{max}, 0:\nu''_{max}$)	Selected ETMF
N ₂	First-positive Second-positive Vegard–Kaplan Wu–Benesch Lyman–Birge–Hopfield IR afterglow	$ \begin{array}{c} B^{3}\Pi_{g} - A^{3}\Sigma_{u}^{+} \\ C^{3}\Pi_{u} - B^{3}\Pi_{g} \\ A^{3}\Sigma_{u}^{+} - X^{1}\Sigma_{g}^{+} \\ W^{3}\Delta_{u} - B^{3}\Pi_{g} \\ a^{1}\Pi_{g} - X^{1}\Sigma_{g}^{+} \\ B^{\prime 3}\Sigma_{u} - B^{3}\Pi_{g} \end{array} $	(0:21,0:21) (0:4,0:21) (0:21,0:21) (0:21,0:17) (0:21,0:21) (0:21,0:21)	[172] [63] [24] [24] [24] [24]
02	Schumann-Runge	$B^3 \Sigma_u^ X^3 \Sigma_g^-$	(0:27,0:27)	[64,65]
N_2^+	Meinel First-negative Second-negative	$A^2 \Pi_u - X^2 \Sigma_g^+ B^2 \Sigma_u^ X^2 \Sigma_g^+ C^2 \Sigma_u^+ - X^2 \Sigma_g^+ $	(0:27,0:27) (0:8,0:21) (0:6,0:21)	[66] [67] [67]
NO	γ β δ ε γ' β' 11,000 Å Infrared	$\begin{array}{l} A^{2} \Sigma + -X^{2} \Pi_{r} \\ B^{2} \Pi_{r} - X^{2} \Pi_{r} \\ C^{2} \Pi_{r} - X^{2} \Pi_{r} \\ D^{2} \Sigma + -X^{2} \Pi_{r} \\ E^{2} \Sigma + -X^{2} \Pi_{r} \\ B^{r2} \Delta - X^{2} \Pi_{r} \\ D^{2} \Sigma + -A^{2} \Sigma + \\ X^{2} \Pi_{r} - X^{2} \Pi_{r} \end{array}$	(0:12,0:22) (0:37,0:22) (0:8,0:22) (0:9,0:22) (0:4,0:22) (0:11,0:22) (0:8,0:8) (0:22,0:22)	[68] [69] [70] [71] [72] [72] [71] [73]
со	Infrared Fourth-positive Hopfield–Birge Third-positive Triplet Asundi Angström	$\begin{array}{l} X^{1} \Sigma^{+} - X^{1} \Sigma^{+} \\ A^{1} \Pi - X^{1} \Sigma^{+} \\ B^{1} \Sigma^{+} - X^{1} \Sigma^{+} \\ b^{3} \Sigma^{+} - a^{3} \Pi_{r} \\ d^{3} \Delta_{i} - a^{3} \Pi_{r} \\ a'^{3} \Sigma^{+} - a^{3} \Pi_{r} \\ B^{1} \Sigma^{+} - A^{1} \Pi \end{array}$	(0:50,0:50) (0:22,0:50) (0:1,0:50) (0:2,0:18) (0:20,0:20) (0:20,0:18) (0:1,0:21)	[74] [75] [77] [78] [79] [80] [77]
CO+	Comet–Tail Baldet–Johnson First-negative	$\begin{array}{l} A^2 \Pi_i - X^2 \Sigma^{ +} \\ B^2 \Sigma^{ +} - A^2 \Pi_i \\ B^2 \Sigma^{ +} - X^2 \Sigma^{ +} \end{array}$	(0:38,0:38) (0:50,0:50) (0:25,0:35)	[81] [81] [81]
CN	Red Violet LeBlanc	$\begin{array}{l} A^2 \Pi_i - X^2 \Sigma^{ +} \\ B^2 \Sigma^{ +} - X^2 \Sigma^{ +} \\ B^2 \Sigma^{ +} - A^2 \Pi_i \end{array}$	(0:38,0:38) (0:27,0:36) (0:25,0:38)	[171] [82] [171]
C ₂	Philips Mulliken Deslandres–D'Azambuja Ballik and Ramsay Swan Fox–Herzberg dc	$\begin{array}{l} A^{1}\Pi_{u} - X^{1}\Sigma_{g}^{+} \\ \mathbf{D}^{1}\Sigma_{u}^{+} - X^{1}\Sigma_{g}^{+} \\ \mathbf{C}^{1}\Pi_{g} - A^{1}\Pi_{u} \\ b^{3}\Sigma_{g}^{-} - a^{3}\Pi_{u} \\ d^{3}\Pi_{g} - a^{3}\Pi_{u} \\ e^{3}\Pi_{g} - a^{2}\Pi_{u} \\ d^{3}\Pi_{g} - c^{3}\Sigma_{u}^{+} \end{array}$	(0:35,0:21) (0:22,0:21) (0:9,0:32) (0:43,0:43) (0:22,0:33) (0:35,0:35) (0:13,0:17)	[83] [84] [85] [83] [83] [85] [83]
H ₂	Lyman Werner	$B^{1}\Sigma_{u}^{+} - X^{1}\Sigma_{g}^{+}$ $C^{1}\Pi_{u} - X^{1}\Sigma_{g}^{+}$	(0:36,0:36) (0:12,0:22)	[86] [86]

Table 1

Electronic systems for which the transition probabilities have been calculated.

J=4.5 are strongly predissociated, leading to shorter radiative lifetimes of $C^2 \Pi_r$ state ($\nu' > 0$). So we don't present comparison with the experimental results. The ETMF selected for δ system is that of Ref. [70].

Concerning the $D^2\Sigma^+$ electronic state of NO, both the ε system and 11,000 Å system contribute to the vibrational radiative lifetimes of NO $D^2\Sigma^+$ electronic state. The radiative lifetimes calculated from the ETMF of Ref. [71] are larger (about 5–18%) than the calculated ones of Ref. [27], although they use the same ETMF. Our calculated results are within the error bars of experimental results for $\nu' = 0$, 3, as shown in Fig. 4(c).

As to the $B'^2\Delta$ electronic state of NO, Fig. 4(d) shows the radiative lifetimes of vibrational levels of the present results, the theoretical values of Ref. [27] and various experimental results of Refs. [89, 118–120]. The ETMF of Ref. [72] has been used in this paper. The different experimental radiative lifetimes are distributed desultorily and our calculated results are bounded by these experimental results.

The infrared system of NO greatly contributes to the discrete radiation of the N_2-O_2 plasma in the infrared region and has been studied thoroughly in the literature. We select the ETMF of Ref. [73] to calculate the Einstein coefficients that agree well (about 1.5%) with the calculated ones of Ref. [27]. Compared to the experimental results [146–151], a deviation of about 9% is observed

at (v', v'') = (1, 0). It should be also noted that a deviation is about 4% in comparison with the experimental results [146,147] at (v', v'') = (2, 0).

On the whole, although experimental radiative lifetimes of the NO $A^2\Sigma^+$, $B^2\Pi_r$, $D^2\Sigma^+$, $B'^2\Delta$ electronic states exhibit a quite large dispersion, our calculated results are bounded by these experimental data.

5.4. O₂ transitions

For O₂ Schumann–Runge band, the ETMF of Ref. [64], shifted by $-0.049 a_0$ as suggested by Friedman [65], is used in this work with the extrapolations of the ETMF from Ref. [65]. As shown in Fig. 5, the absorption oscillator strengths obtained using this ETMF are slightly lower than the calculations of Chauveau et al. [27], whereas the results are between the experimental results from Ref. [121] and Ref. [122] for v' = 0 - 19, v'' = 0, and they are closer to the experimental results [122] for v' = 0 - 19, v'' = 1.

5.5. CO transitions

Due to the great contribution to the discrete radiation of the CO_2-N_2 plasma in the VUV region, the Fourth-positive system of CO have been widely investigated, and different ETMFs have been given in the past few years. Field et al. [123] obtained this ETMF by



Fig. 2. Comparison of the vibrational radiative lifetimes calculated in the present work for N₂, (**a**) $B^3 \Pi_g$ electronic state with the results of Refs. [27]^a (calculation with ETMF of Ref. [45]), [27]^b (calculation with ETMF of Ref. [145]), [91–96], (**b**) $C^3 \Pi_u$ electronic state with the results of Refs. [27], [97]^a (Phase-shift method), [97]^b (Delayed-coincidence method), [98–100], (**c**) $W^3 \Delta_u$ electronic state with the results of Refs. [103,39,15], (**d**) $a^1 \Pi_g$ electronic state with the results of Refs. [101-102].



Fig. 3. Comparison of the vibrational radiative lifetimes calculated in the present work for N₂⁺, (**a**) $A^2 \Pi_u$ electronic state with the results of Refs. [27,104–106], (**b**) $B^2 \Sigma_u^+$ electronic state with the results of Refs. [27,107], (**c**) $C^2 \Sigma_u^+$ electronic state with the results of Refs. [27,108].



Fig. 4. Comparison of the vibrational radiative lifetimes calculated in the present work for NO, (a) $A^2\Sigma^+$ electronic state with the results of Refs. [27,68,90,109–113], (b) $B^2\Pi_r$ electronic state with the results of Refs. [27,89,109,114–116], (c) $D^2\Sigma^+$ electronic state with the results of Refs. [27,89–90,112–113,117–118], (d) $B'^2\Delta$ electronic state with the results of Refs. [27,89,109,114–116], (c) $D^2\Sigma^+$ electronic state with the results of Refs. [27,89–90,112–113,117–118], (d) $B'^2\Delta$ electronic state with the results of Refs. [27,89,109,114–116], (e) $D^2\Sigma^+$ electronic state with the results of Refs. [27,89–90,112–113,117–118], (e) $D^2\Delta$ electronic state with the results of Refs. [27,89–109,114–116], (e) $D^2\Sigma^+$ electronic state with the results of Refs. [27,89–90,112–113,117–118], (e) $D^2\Delta$ electronic state with the results of Refs. [27,89–109,114–116], (e) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (e) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (e) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116], (f) $D^2\Delta^-$ electronic state with the results of Refs. [27,89–109,114–116],



Fig. 5. Comparison of absorption oscillator strengths calculated in the present work for O_2 Schumann–Runge band with the results of Refs. [27,121,122]

measuring radiative lifetimes using synchrotron radiation. DeLeon [75,76] deduced the ETMF by using the laser induced fluorescence. And Kirby and Cooper [77] calculated this value through the "abinitio" method. We select the ETMF of DeLeon which enables to obtain a good agreement with the experimental radiative lifetimes of Ref. [123], just as shown in Fig. 6(a). The calculated radiative lifetimes are relatively dispersed, but the present results are among other calculated values.

For the Third-positive system, there is no "ab-initio" ETMFs for us to use in the references that we can find. Hence, we adopt the fitting formula from Ref. [26] obtained from experimental lifetimes of Carlson [78]. Table 2 shows the vibrational radiative lifetimes of the $b^3 \Sigma^+$ electronic state. The radiative lifetimes obtained using

Table 2
Vibrational radiative lifetimes of the $b^3 \Sigma^+$ electronic state of
CO: comparison with the experimental results.

	This work	Experimental results					
		Ref. [152]	Ref. [124]	Ref. [78]			
$\begin{array}{l} \nu' = 0 \\ \nu' = 1 \end{array}$	57.8 62.6	57.6±1.24	$\begin{array}{c} 56{\pm}1\\ 56{\pm}1 \end{array}$	53.7±1.5 64.6±2			

this ETMF are in good agreement with the experimental results of Ref. [152] for v' = 0, and they are closer to the experimental data of Ref. [78] for v' = 1.

For the Triplet system, the only ETMF that we can find is from Ref. [79], and the ETMF was calculated from the only equation in Ref. [79] using the measured radiative lifetimes, concluding with a constant of Re = 1.65 Debye.

Considering the Asundi system, we select the ETMF of Peterson and Woods [80] that are calculated with CASSCF wave functions rather than with CI-SD wave functions since it covers a large valid range of internuclear distance. The calculated radiative lifetimes are shown in Fig. 6(b) and a good agreement with calculations of da Silva and Dudeck [26] is observed. However, our calculated results are quite larger than the only experimental radiative lifetimes of Ref. [124] we can find in the literature. The deviation decreases from 23% for v' = 4 to 11% for v' = 9 with the increasing v' value.

Table 3 gives the vibrational radiative lifetimes of $B^1\Sigma^+$ electronic state with comparison to the theoretical and experimental results. Both the Hopfield–Birge system and Angström system are taken into consideration here, and the ETMFs of Ref. [77] are used for these two electronic transition systems. By comparison, a good



Fig. 6. Comparison of the vibrational radiative lifetimes calculated in the present work for CO, (a) $A^1\Pi$ electronic state with the results of Refs. [26,28,77–78,123], (b) $a'^3\Sigma^+$ electronic state with the results of Refs. [26,124].

Table 3 Vibrational radiative lifetimes of the $B^1\Sigma^+$ electronic state of CO: comparison with the theoretical and experimental results.

	This work	Theoretical results Ref. [77]	Experimental results Ref. [153]
$\begin{array}{l} \nu' = 0 \\ \nu' = 1 \end{array}$	32.5	33.5	24.3±1.8
	29.4	29.2	23.8±1.4



Fig. 7. Comparison of the vibrational radiative lifetimes calculated in the present work for CN, (a) $A^2 \Pi_i$ electronic state with the results of Refs. [26,28,82,125–129], (b) $B^2 \Sigma^+$ electronic state with the results of Refs. [26,28,82,125–128].

agreement (of about 3%) is observed between the present radiative lifetimes and the calculated results from Ref. [77] for v' = 0 and a better agreement (of about 0.5%) is observed for v' = 1. Whereas our results deviate greatly from the experimental results measured by Rogers and Anderson [153].

5.6. CN transitions

Fig. 7(a) presents the vibrational radiative lifetimes of the $A^2 \Pi_i$ electronic state of CN. As shown, the vibrational radiative lifetimes calculated using the ETMF of Ref. [171] are in good agreement with most of the theoretical results [28,82,126], although the radiative lifetimes of da Silva and Dueck [26] calculated utilizing the ETMF of Ref. [155] appear slightly higher than the other calculated results. As shown in Fig. 7(a), large deviations are observed between the calculated values and the experimental results. Besides, the distribution of different experimental radiative lifetimes is so disorderly that the difference between two experimental results for

the same vibrational level is up to more than 50% relative to the larger radiative lifetime. Therefore, we assume that the theoretical results are more convincing, just as pointed out by Kuznetsova and Stepanov [154].

Fig. 7(b) illustrates the vibrational radiative lifetimes of the $B^2\Sigma^+$ electronic state of CN. One of them is obtained by experimental measurement and the others are derived from calculations. As shown, our calculated radiative lifetimes are in very good agreement with theoretical results from Refs. [26,28,82,126] except those of Ref. [125]. Compared to the only experimental results measured by Duric et al. [128] we can find in the literature, a good agreement is observed for v' = 2, 3, 4, 5 and a large discrepancy of about 7% is observed for v' = 0, 1. Note that the Violet system and the LeBlanc system are both considered to contribute to the radiative lifetimes of CN $B^2\Sigma^+$ electronic state. As for these two electronic transition systems, Knowles et al. [82] and Bauschlichler et al. [155] give two different sets of ETMFs, which both provide a good reproduction of the experimental radiative lifetimes. Here we



Fig. 8. Comparison of the vibrational radiative lifetimes calculated in the present work for C_2 , (**a**) $A^1 \Pi_u$ electronic state with the results of Refs. [22,26,28,85,130–133], (**b**) $b^3 \Sigma_g^-$ electronic state with the results of Refs. [26,28,83,85], (**c**) $d^3 \Pi_g$ electronic state with the results of Refs. [26,28,133–137], (**d**) $D^1 \Sigma_u^+$ electronic state with the results of Refs. [85,138].

select the ETMFs of Ref. [82]. It also should be noted that the results in Fig. 7(b) by da Silva and Dudeck exclude the LeBlanc system.

5.7. C₂ transitions

For the Philips, Ballik and Ramsay, Swan and $d^3\Pi_g - c^3\Sigma_u^+$ systems, we select the latest "ab-initio" ETMFs of Ref. [83] because they cover a wider range of internuclear distances than those of other references. In the case of Philips system, it has been widely studied theoretically and experimentally. Fig. 8(a) shows the vibrational radiative lifetimes of the $A^1\Pi_u$ electronic state of C₂. As shown, the present calculated radiative lifetimes are in quite good agreement (of about 4%) with calculations of Refs. [22,26,28,130] except those of Ref. [85]. However, our results are significantly lower than available experimental results of Refs. [131–133], though the radiative lifetimes for v' = 0, 1, 5–8 are inside the error bars of Ref. [132]. for the Ballik and Ramsay system. Fig. 8(b) gives some theoretical radiative lifetimes of the $b^3 \Sigma_{\sigma}^$ electronic state of C₂. A very good agreement (within 4%) between the present radiative lifetimes and the ones calculated by Kokkin et al. [83] is observed for v' = 0, 1, 2. Nevertheless, discrepancies of about 36% (or 88%) remain between our calculated results and theoretical results reported in Ref. [28,85] (or Ref. [26]) relative to present radiative lifetimes. As to the two other systems, they both contribute to vibrational radiative lifetimes of $d^3 \Pi_g$ electronic state of C₂ and the detailed comparisons between the present radiative lifetimes and various results in the literature are given in Fig. 8(c). Moreover, a large deviation among different experimental results can be seen and our results are in good agreement with the values of Ref. [133-135]. It should also be noticed that our results are in excellent agreement with the calculations of Ref. [28], although they deviate greatly from the calculations of Ref. [26].

Concerning the Mulliken system, the theoretical ETMF of Bruna and Wright [84] is used in this paper. The radiative lifetimes calculated from this ETMF are slightly higher than the experimental results of Ref. [138], but inside the error bars, just as shown in Fig. 8(d). And a slight higher difference (about 5%) can be found in our calculated results compared to the theoretical results from Ref. [85].

For the Deslandres–D'Azambuja system, we use the ETMF of Ref. [85] rather than that of Cooper and Nicholls [156,157], as recommended by Lino da Silva and Dudeck [36], who consider that the results of Ref. [85] are more accurate. However, there is a deviation of 6% between this ETMF and experimental value [85].

As for the Fox–Herzberg system, Chabalowski et al. [85] and Cooper et al. [156] give two ab-initio ETMFs, which are very close. The ETMF of Ref. [85] is chosen in this paper because it covers a wider range of internuclear distances.

5.8. CO⁺ transitions

For Comet–Tail, Baldet–Johnson and First-negative systems, we select the ETMFs of Ref. [81], which enable to obtain a good agreement with the available experimental radiative lifetimes, just as shown in Fig. 9. For the $A^2\Pi_i$ electronic state, the calculated radiative lifetimes are in quite good agreement with the theoretical results of Ref. [28] and the experimental results of Refs. [139,140]. For the $B^2\Sigma^+$ electronic state, a very good agreement (of about 1%) between our results and the calculations of Ref. [28] is observed for v' = 0-2, whereas the radiative lifetime at v' = 3 of Ref. [28] is quite larger than the present result and experimental results of Refs. [141–143]



Fig. 9. Comparison of the vibrational radiative lifetimes calculated in the present work for CO⁺, (a) $A^2\Pi_i$ electronic state with the results of Refs. [28,139–140], (b) $B^2\Sigma^+$ electronic state with the results of Refs. [28,139–143].



Fig. 10. Comparison of the vibrational radiative lifetimes calculated in the present work for H₂, (a) $B^1 \Sigma_u^+$ electronic state with the results of Refs. [173,158–162], (b) $C^1 \Pi_u$ electronic state with the results of Refs. [158–160,173].

5.9. H₂ transitions

For the two electronic transition systems of H₂ considered here, we use the ETMFs given by Drira et al. [86]. Vibrational radiative lifetimes of the $B^1 \Sigma_u^+$ and $C^1 \Pi_u$ electronic states are calculated with the ETMFs and our vibrational wave functions. Comparisons between our calculated results and experimental data are shown in Fig. 10(a) for the $B^1 \Sigma_u^+$ electronic state. A quite good agreement between the calculated radiative lifetimes and the ones measured by Hesser et al. [160] and Schmoranzer et al. [162] is observed, while a large discrepancy of about 33% is observed for v' = 9-11. This discrepancy was explained by Fantz et al. [173] who believed that different input spectroscopic constants and ETMFs will lead to large deviations for higher vibrational levels. As shown, the latest theoretical results of Fantz et al. [144,173] are also chosen to validate our calculated results and an excellent agreement is noticed in Fig. 10(a). For the $C^1 \Pi_u$ electronic state, our calculated radiative lifetimes agree well with some theoretical results [158,159,173], but a large discrepancy is observed for v' = 0-3 compared to the only experimental results [160] we can find in the literature, as shown in Fig. 10(b).

6. Conclusions

In conclusion, we have provided more detailed and accurate tables of the radiative transition probabilities for the main diatomic electronic transition systems contributing to the radiative processes encountered in plasmas produced in Earth, Mars and Jupiter atmospheric entry. The main electronic systems of N_2 , N_2^+ , NO, O₂, CO, CO⁺, CN, C₂ and H₂ have been investigated. The potential energy curves of each electronic state of interest were reconstructed using the RKR inversion procedure based on the most upto-date spectroscopic constants. Then the radial Schrödinger equation was solved to obtain the vibrational wave functions. These results, together with the latest published "ab-initio" ETMFs were used to calculate some tables of radiative transition probabilities for the main molecular radiative transitions, which are provided in the supplementary material. These arrays can be used to compute the spectral emission and absorption coefficients for the diatomic bound-bound transitions. The radiative lifetimes can also be used to research the collisional-radiative processes in nonequilibrium plasmas.

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Appendix A. Selected spectroscopic constants

Table A.1

Klein–Dunham coefficients for the selected states of the N₂ molecule (all values are in cm⁻¹ except r_e in Angstrom and v_{max}).

_												
		Y _{ij}	0	1	2	3	4	5	6	Ediss	r _e	v_{max}
	$X^{3}\Sigma_{\sigma}^{+}$ [37,163]	0	0.0	2358.56	14.317	-3.31E-3	-1.95E-4			79886.6	1.097685	15
	8	1	1.998236	1.730986E-2	-3.01203E-5	-6.927E-8						
		2	5.73729E-6	8.951E-9								
	$A^{3}\Sigma_{u}^{+}$ [38,163]	0	50203.6	1460.94	13.98	2.4E-2	-2.56E-3			30135.8	1.2866	16
		1	1.4539	1.75E-2	-1.4E-4							
		2	5.46E-6	1.1E-7								
	$B^3 \Pi_g$ [38,163]	0	59619.3	1734.025	14.412	-3.3E-3	-7.9E-4	4.2E-5	-1.68E-6	39811.7	1.2126	21
		1	1.63772	1.793E-2	-1.0E-4	5.0E-6	-2.1E-7					
		2	5.88E-6	1.3E-8								
	$W^{3}\Delta_{u}$ [39,163]	0	59808	1506.5	12.5					39301	1.2784	17
		1	1.47	1.71E-2								
	$B'^{3}\Sigma_{u}$ [40,163]	0	66272.4	1516.6	12					42452	1.2784	21
		1	1.473	1.666E-2								
		2	5.55E-6									
	$C^{3}\Pi_{u}$ [41,163]	0	89136.9	2047.79	28.942	2.25	-0.551			10140.7	1.14869	4
		1	1.8268	2.4E-2	1.9E-3	-6.0E-4						
		2	5.1E-6	2.2E-6	-1.3E-6	2.4E-7						
	$a^1 \Pi_g$ [40,163]	0	69283.1	1694.2	13.949					49051	1.2203	10
	-	1	1.6169	1.793E-2								
		2	5.89E-5									

Table A.2

Klein–Dunham coefficients for the selected states of the N₂⁺ molecule (all values are in cm⁻¹ except r_e in Angstrom and v_{max}).

	Y _{ij}	0	1	2	3	4	5	6	7	Ediss	r _e	$v_{\rm max}$
$X^2 \Sigma_{\sigma}^+$ [36,164]	0	0.0	2197.95	16.0906	4.0E-3	-6.1E-3	3.9E-4	1.4E-5	2.0E-7	71368	1.1191	21
8	1	1.92286	1.8634E-2	-6.77E-5	-2.32E-6							
	2	5.92E-6	3.9E-8									
$A^2 \Pi_u$ [36,164]	0	9167.46	1900.73	14.9618	1.12E-2	-0.27E-3				62352	1.1772	27
	1	1.73539	1.8652E-2	-6.0E-5	-1.1E-6							
	2	5.6E-6										
$B^2 \Sigma_u^+$ [36,164]	0	25461.11	2406.67	23.3782	-0.3	-6.67E-2				45799	1.0771	4
	1	2.07492	2.088	-5.0E-4	-8.8E-5							
	2	6.17E-6										
$C^2 \Sigma_u^+$ [36,165]	0	64609.05	2069.4	-8.3	1.6E-2					26057.15	1.2628	6
	1	1.5098	-1.0E-3									

Table A.3

Klein–Dunham coefficients for the selected states of the NO molecule (all values are in cm⁻¹ except r_e in Angstrom and v_{max}).

	Y _{ij}	0	1	2	3	4	5	6	7	Ediss	r _e	$v_{\rm max}$
X ² Π _r [42]	0	0.0	1904.13	14.08836	1.00467E-2	-1.5331E-4	-9.769E-6	-1.9142E-7	5.2734E-9	53344	1.15077	22
	1	1.704888	0.01754158	1.4886E-5		-4.7275E-8	1.0108E-9	-6.0557E-11				
	2	5.466E-6	1.7032E-8									
$A^2 \Sigma^+$ [40]	0	43965.7	2374.31	16.106	4.65E-2					28603	1.0634	8
	1	1.9965	1.915									
	2	5.4E-6										
$B^2 \Pi_r$ [43]	0	45932.3	1042.4	7.7726	0.11596	-3.9577e-3				26637	1.4167	28
	1	1.1244	0.013433	2.991Ee-5	3.177E-6							
	2	5.2E-6	1.4E-8									
$C^2 \Pi_r$ [43]	0	52179.8	2381.3	15.702	7.253E-2					20383	1.062	9
	1	2.0155	3.244E-2									
	2	5.8E-6	1.8E-8									
$D^2 \Sigma^+$ [40]	0	53084.7	2323.9	22.885	0.75	-0.22				19484	1.0618	5
	1	2.0026	2.175									
	2	5.9E-6	5.9E-8									
$B'^{2}\Delta$ [44]	0	60364.2	1217.4	15.61						12205	1.302	6
	1	1.332	2.1E-2									
$E^2 \Sigma^+$ [40]	0	60628.8	2375.3	16.43						11940	1.06615	4
	1	1.9863	1.82E-2									
	2	5.6E-6										

Table A.4

Klein–Dunham coefficients for the selected states of the O₂ molecule (all values are in cm⁻¹ except r_e in Angstrom and v_{max}).

	Y _{ij}	0	1	2	3	4	5	6	7	Ediss	r _e	$v_{\rm max}$
$X^3 \Sigma_g^-$	0	0.0	1580.1932	11.980804	0.047474736	-1.72748E-3				42048	1.20752	19
[23,163]	1	1.445622	0.01593268	6.406456E-5	-2.846158E-6							
	2	4.839E-6	2.06E-6									
$B^3 \Sigma_u^-$	0	49793.3	709.0577	10.61408	-0.059212435	-0.023974994	2.2067951E-3	-1.5990957E-4	4.4274814E-6	8556.5	1.60426	21
[35,163]	1	0.818975	0.0119225	-6.30472E-4	1.57426E-6	6.70586E-6	-9.35318E-7	2.901E-8				
	2	4.55E-6	2.2E-7									

Table A.5	
Klein–Dunham coefficients for the selected states of the CO molecule (all values are in \mbox{cm}^{-1}	except $r_{\rm e}$ in Angstrom and $v_{\rm max}$).

	Y _{ij}	0	1	2	3	4	5	6	7	8	9	Ediss	r _e	$v_{\rm max}$
$X^1\Sigma^{+}$	0	0.0	2169.81267	13.2878763	1.04111E-2	6.93664E-5	1.67935E-7	2.05925E-9	-8.488E-10	1.2388E-11	-8.234E-14	90575.3	1.1327	41
[45,166]	1	1.93128098	1.75044E-2	7.17392E-7	-2.1464E-8	4.4354E9	-1.361E-10	1.2458E-12	-2.125E-14					
	2	6.12162E-6	-1.0349E-9	1.8498E-10	-2.431E-12									
$a^3 \Pi_r$	0	48686.8	1743.41	14.36	-4.5E-2	2.5E-5						41888.6	1.2105	12
[46,76]	1	1.69124	1.904	-4.1E-5										
	2	6.36E-6	4.0E-8											
$a'^3\Sigma^+$	0	55825.5	1228.6	10.466	9.1Ee-3	2.59E-3						34842.9	1.3523	9
[40]	1	1.3446	1.892E-2											
	2	6.41E-6												
$d^3\Delta_i$	0	61120.1	1171.94	10.635	7.85E-2	-1.63E-5						29519	1.3740	10
[40,76]	1	1.3108	1.782E-2	1.13E-4										
	2	6.59E-6												
$A^1\Pi$	0	65075.8	1518.24	19.4	0.76584	-0.14117	0.01434	-8.051E-4	2.36E-5	2.9E-7		25499.6	1.2408	19
[47,168]	1	1.6115	0.023251	1.5911E-3	-5.716E-4	8.2417E-5	-5.9413E-6	2.1149E-7	2.991E-9					
	2	7.29E-6	1.05E-7											
$B^1\Sigma^+$	0	86932.16	2150.41	34.09								3643.14	1.1197	2
[48]	1	1.96103	2.574E-2											
	2	6.477E-6	5.86E-7											
$b^3\Sigma^+$	0	83686.5	2333.9	58.64								5908.5	1.113	2
[49]	1	1.986	4.2E-2											

Table A.6

Klein–Dunham coefficients for the selected states of the CO⁺ molecule (all values are in cm⁻¹ except r_e in Angstrom and v_{max}).

	Y _{ij}	0	1	2	3	Ediss	r _e	$v_{\rm max}$
$X^2\Sigma^+$ [56,167]	0	0.0	2214.127	15.094	-1.17E-2	68288.8	1.11514	5
	1	1.976941	1.8943E-2	-3.44E-6				
	2	6.313E-6	1.53E-8					
A ² Π _i [56,167]	0	207332.037	1561.806	13.4785	8.65Ee-3	47556.8	1.24377	8
	1	1.589392	1.9494E-2	-1.17E-7				
	2	6.729E-6	-3.84E-8					
$B^2\Sigma^+$ [57,81]	0	45876.7	1734.57	28.248	0.399	38279.6	1.16877	2
	1	1.7999	3.025E-2					
	2	7.75E-6	2.2E-7					

Table A.7

Klein–Dunham coefficients for the selected states of the CN molecule (all values are in cm^{-1} except r_e in Angstrom and v_{max}).

	Y _{ij}	0	1	2	3	4	5	6	7	Ediss	r _e	$v_{\rm max}$
$X^2 \Sigma^+$	0	0.0	2068.6834	13.12172	-5.412E-2	-9.83E-5	3.23E-5			63296.6	1.1718073	11
[54,169]	1	1.8997835	0.01737487	-2.43E-5	-4.92E-7	-1.61E-8						
	2	6.41E-6	1.2E-8									
$A^2 \Pi_i$	0	9240	1813.26	12.76873	-3.569E-3	1.070745E-4	-4.6648E-6	-2.5273E-7		54053.3	1.2333	20
[59]	1	1.71591	1.716732E-2	-5.4905E-5	6.759374E-6	5.88513E-7	1.890441E-8	-2.34754E-10				
	2	5.93E-6	4.2E-8									
$B^2 \Sigma^+$	0	25752	2161.46	18.219	-0.486	0.04	-6.93E-3	4.32E-4	-8.34E-6	56777	1.150	17
[60,169]	1	1.96891	2.0337E-2	-5.9E-5	-7.71E-5	3.1E-6						
	2	6.5322E-6	8.0725E-8									

Table A.8

				-		= 1		-		
	Y _{ij}	0	1	2	3	4	5	Ediss	r _e	$v_{\rm max}$
$X^{1}\Sigma_{\sigma}^{+}$ [50,170]	0	0.0	1855.01	13.555	-0.132	3.57E-3	-1.116E-3	50247.9	1.2425	6
8	1	1.8201	1.801E-2	-6.33E-5	-2.06E-6					
	2	6.964E-6	6.41E-8							
$a^{3}\Pi_{u}$ [51,163]	0	716.2	1641.35	11.670				49531.7	1.3119	9
	1	1.6342	1.661							
	2	6.44E-6								
$b^{3}\Sigma_{g}^{-}$ [52,163]	0	6434.8	1470.4	11.155	1.39E-2			43812	1.36928	6
Ŭ.	1	1.4986	1.631E-2	-4.61E-6						
	2	6.1958E-6	6.62E-9							
$A^1 \Pi_u$ [53,170]	0	8391.3	1608.2	12.055	-0.012			41867	1.3184	8
	1	1.6165	1.687E-2	-5.47E-5						
	2	6.494E-6	3E-8							
$c^{3}\Sigma_{u}^{+}$ [40,163]	0	9124.2	2085.9	18.623				37988.4	1.23	9
	1	1.921	1.255E-2							
$d^{3}\Pi_{g}$ [54,163]	0	20022.5	1788.22	16.457	-0.501			30225.4	1.2661	10
	1	1.75523	1.907E-2	5.35E-4						
	2	6.74E-6								
$e^3 \Pi_g$ [55,163]	0	40796.7	1106.56	39.26	2.81	-0.127		19643.9	1.5351	4
	1	1.1922	2.42E-2							
	2	6.3E-6								
$D^{1}\Sigma_{g}^{+}$ [40,163]	0	43239.8	1829.57	13.94				39843.5	1.2380	3
-	1	1.8332	1.96E-2							
	2	7.32E-6								

Klein–Dunham coefficients for the selected states of the C₂ molecule (all values are in cm⁻¹ except r_e in Angstrom and v_{max}).

Table A.9

Klein–Dunham coefficients for the selected states of the H₂ molecule (all values are in cm⁻¹ except r_e in Angstrom and v_{max}).

	Y _{ij}	0	1	2	3	4	5	6	7	8	E _{diss}	r _e	$v_{\rm max}$
$X^1 \sum_{g}^+$	0	0.0	4432.29142	-149.2039	12.01408	-2.369503	0.2852839	-0.020446	7.82E-04	-1.27E-05	38163.37	0.74152	14
[61]	1	61.264895	-4.274626	1.0029705	-0.329079	0.057094	-5.38E-03	2.59E-04	-5.04E-06				
	2	-0.049649	0.0106961	-6.00E-03	1.64E-03	-2.18E-04	1.38E-05	-3.35E-07					
	3	5.80E-05	-2.84E-05	1.46E-05	-3.05E-06	2.73E-07	-8.75E-09						
	4	-7.00E-08	4.25E-08	-1.69E-08	2.39E-09	-1.08E - 10							
	5	6.39E-11	-3.54E-11	9.45E-12	-6.70E-13								
	6	-3.81E-14	1.54E-14	-2.06E-15									
$B^1 \sum_{u}^+$	0	91528.4	1357.19	-20.15	0.46	-0.021					28767.93	1.2894	6
[62]	1	19.984	-1.115	0.0836	-0.0044								
	2	-0.01656	2.08E-3	-0.00013									
C¹∏u	0	100089.8	2443.77	-69.524	0.7312	-0.0415					20236.8	1.03346	13
[62]	1	31.3629	-1.6647	0.0296	-0.00296								
	2	-0.0223	0.00074										

Appendix B. Supplementary material (in .CSV) files

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jqsrt.2017.08.010. The tables of supplementary material give the Einstein coefficients in s^{-1} , absorption band oscillator strengths and Franck–Condon factors corresponding to the upper v' vibrational levels in columns and the lower v'' vibrational levels in rows.

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